

APPROXIMATE METHOD FOR CALCULATING TURBULENT TWO-PHASE FLOW IN A CHANNEL WITH PERMEABLE WALLS

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UDC 532.529

Based on a model of a double-velocity and two-temperature medium the authors constructed a system of equations that describes plane or axisymmetric turbulent flow of a gas suspension in a channel with permeable walls. The system of equations of motion and heat transfer reduces to a system of ordinary differential equations, whose integration is much less difficult than solution of the initial system. The authors obtained the distribution of the velocity and local characteristics of turbulence in the channel with injection with allowance for the inverse effect of a condensed phase.

Introduction. Internal flows are usually realized in the region of high Reynolds numbers that correspond to a turbulent regime of flow characterized by dependence of the viscosity and the Prandtl number on local characteristics of the flow. As a result of experimental investigations of the characteristics of turbulent flows in channels with uniform injection [1, 2] it was shown that at a distance from the inlet to the channel a quasistabilized regime of flow is established when the parameters of the flow normalized to the local velocity depend weakly on the axial coordinate. This was confirmed subsequently by other experiments [8] and calculations based on two-parameter turbulence models [4, 5].

In evaluating a number of flow characteristics, two-parameter turbulence models yield results that are in better agreement with the data of physical experiment than classical semiempirical models, for example, the Prandtl model or its later modifications that allow for the effect of a transverse mass flux on the mechanisms of turbulent transfer [6]. In [4], the phenomenon of laminarization of a turbulent flow in a channel with injection found experimentally in [1, 3] was reproduced theoretically based on a $k-\varepsilon$ turbulence model.

At the same time, simpler models whose construction involves a rational simplification of the initial system of equations, such as [2, 5], are also being developed. In particular, in [5], the distributions of local characteristics of turbulence over the cross section of a channel are obtained in final form based on a preliminary evaluation of the order of smallness of different terms.

In practice, the intensification of transfer properties of a medium is largely attributable to the presence of condensed-phase particles, which brings up the necessity of calculating the characteristics of two-phase flows in channels with permeable walls.

Approaches intended for description of turbulent flows of a gas suspension have been investigated rather thoroughly [7, 8]. However, the creation of simplified models that allow for special features of the flows formed and the character of mass supply from the channel walls is required to reproduce the specific regimes of flows in channels with injection, to obtain direct numerical evaluations of the flow characteristics, and to work out recommendations that make the construction of models of three-dimensional flows easier. In the available works, either the motion of particles in a known gasdynamic field without allowance for their inverse effect is considered or the carrying flow is assumed to be laminar [9-11].

1. Formulation of the Problem. Let us consider quasideveloped flow in a long channel when the characteristics of the flow referred to the velocity on the axis of the channel vary slightly along its length. The condition of the existence of the quasideveloped flow can be written in the following form:

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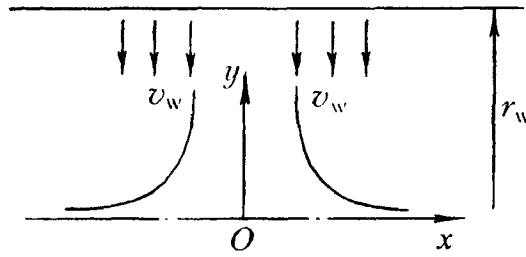


Fig. 1. Flow in a channel with permeable walls.

$$\frac{r_w}{u_m} \left| \frac{du_m}{dx} \right| \ll 1 \quad \text{or} \quad \frac{|v_w|}{u_m} \ll 1.$$

In injection, this flow is always established in channels of sufficient length behind the region of the inlet section [1].

We bring the axis x into coincidence with the plane or the axis of symmetry of the channel and guide the axis y perpendicularly to it (Fig. 1). The cross dimension of the channel r_w is assumed to be constant along the entire length while the injection rate v_w is assumed to be the same at all points on the permeable surface of the channel and to be directed normally to it. The liquid is taken to be spreading symmetrically relative to the plane $x = 0$.

2. Basic Equations. A double-velocity and two-temperature model of interpenetrating continua is used for calculation of the characteristics of two-phase flow.

The flow is assumed to be quasistationary and is described within the framework of a model of an incompressible medium. The condensed phase is modeled by a continuum devoid of intrinsic stresses. The particles are spheres of the same diameter; their collisions are disregarded. In the model of the interaction of a particle with a carrying flow, allowance is made only for the force of hydrodynamic resistance. The coefficient of the resistance is calculated according to the Stokes law $C_D = 24/Re_p$, where $Re_p = |v_g - v_p| d_p / \nu$ is the Reynolds number in the relative motion of the phases.

The averaged continuity, momentum, and energy equations for the gas and dispersed phases accurate to the correlation moments of the third order have the form:

$$\frac{\partial \langle v_{gk} \rangle}{\partial x_k} = 0; \quad (1)$$

$$\langle v_{gk} \rangle \frac{\partial \langle v_{gi} \rangle}{\partial x_k} = -\frac{1}{\rho_g} \frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_k} \left(\nu \frac{\partial \langle v_{gi} \rangle}{\partial x_k} \right) - \frac{\partial \langle v'_{gi} v'_{gk} \rangle}{\partial x_k} - S_{v_i}; \quad (2)$$

$$\langle v_{gk} \rangle \frac{\partial \langle \vartheta_g \rangle}{\partial x_k} = \frac{\partial}{\partial x_k} \left(\frac{\nu}{Pr} \frac{\partial \langle \vartheta_g \rangle}{\partial x_k} \right) - \frac{\partial \langle v'_{gk} \vartheta'_g \rangle}{\partial x_k} - S_{\vartheta}; \quad (3)$$

$$\frac{\partial}{\partial x_k} (\langle \rho_p \rangle \langle v_{pk} \rangle + \langle \rho'_p v'_{pk} \rangle) = 0; \quad (4)$$

$$(\langle \rho_p \rangle \langle v_{pk} \rangle + \langle \rho'_p v'_{pk} \rangle) \frac{\partial \langle v_{pi} \rangle}{\partial x_k} = \frac{\partial}{\partial x_k} (-\langle \rho_p \rangle \langle v'_{pk} v'_{pi} \rangle - \langle v_{pk} \rangle \langle \rho'_p v'_{pi} \rangle) + \rho_g S_{v_i}; \quad (5)$$

$$c_0^m (\langle \rho_p \rangle \langle v_{pk} \rangle + \langle \rho'_p v'_{pk} \rangle) \frac{\partial \langle \vartheta_p \rangle}{\partial x_k} = \frac{\partial}{\partial x_k} (-\langle \rho_p \rangle \langle v'_{pk} \vartheta'_p \rangle - \langle v_{pk} \rangle \langle \rho'_p \vartheta'_p \rangle) + \rho_g S_{\vartheta}. \quad (6)$$

Summation over the recurrent subscripts is assumed.

The inverse effect of the dispersed phase due to the interphase velocity and temperature slip is described by the terms:

$$S_{v_i} = \frac{\rho_p}{\rho_g} \frac{\langle v_{gi} \rangle - \langle v_{pi} \rangle}{\tau_v}; \quad S_{\vartheta} = \frac{\rho_p}{\rho_g} \frac{c_p^m \langle \vartheta_g \rangle - \langle \vartheta_p \rangle}{\tau_{\vartheta}}.$$

In system of Eqs. (1)-(6) the terms that describe the release of dissipative heat due to the work of viscous forces and the forces of interphase interaction and the terms that contain the pulsations of the transport and pressure coefficients are omitted. The correlation moment of pulsations of the discrete-component concentration and velocity are disregarded because of their insignificant role in calculating the characteristics of flows formed by injection [6].

For representation of the components of the turbulent-stress tensor, we adopt the generalized Kolmogorov-Boussinesq hypothesis and the concept of turbulent viscosity, to calculate which the Kolmogorov-Prandtl formula $\nu_t = c_{\mu} k^2 / \varepsilon$ is used. The differential transport equations of the second moments of pulsations of the velocity of the carrying turbulent flow are replaced by the algebraic expressions

$$\langle v'_{gi} v'_{gi} \rangle = -\nu_t \left(\frac{\partial \langle v_{gi} \rangle}{\partial x_j} + \frac{\partial \langle v_{gj} \rangle}{\partial x_i} - \frac{2}{3} \frac{\partial \langle v_{gk} \rangle}{\partial x_k} \delta_{ij} \right) + \frac{2}{3} k \delta_{ij}.$$

Instead of the transport equation of the correlation moments of gas velocity and temperature pulsations, to calculate the turbulent heat flux, we use an expression written in the form of the Fourier law:

$$\langle v'_{gi} \vartheta'_{g} \rangle = -\frac{\nu_t}{Pr_t} \frac{\partial \langle \vartheta_g \rangle}{\partial x_i},$$

in which the turbulent Prandtl number is assumed to be constant.

The relations for the kinetic turbulence energy and its dissipation rate

$$\langle v_{gk} \rangle \frac{\partial k}{\partial x_k} = \frac{\partial}{\partial x_k} \left[\left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_k} \right] + P - \varepsilon - S_k; \quad (7)$$

$$\langle v_{gk} \rangle \frac{\partial \varepsilon}{\partial x_k} = \frac{\partial}{\partial x_k} \left[\left(\nu + \frac{\nu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial x_k} \right] + c_{\varepsilon 1} \frac{\varepsilon}{k} P - c_{\varepsilon 2} \frac{\varepsilon^2}{k} - S_{\varepsilon}. \quad (8)$$

are used in addition to the system of Eqs. (1)-(6). The term P in Eqs. (7)-(8) describes the generation of turbulence. Near the wall, account is taken of the coefficients of the model of turbulent transfer c_{μ} , $c_{\varepsilon 1}$, and $c_{\varepsilon 2}$ as functions of the turbulent Reynolds number $Re_t = k^2 / \nu \varepsilon$. The wall functions are calculated by the formulas of [5]. The curvature of the streamlines is allowed for by the introduction of functions dependent on the turbulent Richardson number [7].

The use of a low-Reynolds version of the two-parameter $k-\varepsilon$ model of turbulence for calculating pulsation characteristics enables us to allow for the curvature of the streamlines and the effects of laminarization of the turbulent flow under the action of injected gas that are characteristic of flows in the channels with permeable walls.

The equations of the model of turbulence as compared to single-phase flow contain additional terms that allow for the inverse effect of particles [7, 8]:

$$S_k = \frac{\rho_p}{\rho_g \tau_v} (\langle v'_{gk} v'_{gk} \rangle - \langle v'_{gk} v'_{pk} \rangle); \quad S_{\varepsilon} = 2\nu \frac{\rho_p}{\rho_g \tau_v} \left\langle \frac{\partial (v'_{gi} - v'_{pi})}{\partial x_k} \frac{\partial v'_{gi}}{\partial x_k} \right\rangle.$$

To model the motion of hydrodynamically fine particles, we restrict ourselves to the construction of a locally homogeneous approximation to the method of space-time averaging [8]. Disregarding, for fine particles, the convective and diffusion terms in the equations of transport of the second single point moments of condensed-phase velocity pulsations [7], we obtain the following relations

$$\langle v'_{pi} v'_{pj} \rangle = f_{vv} \langle v'_{gi} v'_{gj} \rangle; \quad \langle v'_{pi} \vartheta'_p \rangle = \frac{\tau_v f_{v\vartheta} + \tau_\vartheta f_{\vartheta v}}{\tau_v + \tau_\vartheta} \langle v'_{gi} \vartheta'_g \rangle. \quad (9)$$

for the correlation moments of the gas and dispersed phases. The exactness of the given formulas increases as the characteristic times of relaxation of a particle decreases. In view of (9) we have:

$$S_k = 2 \frac{\rho_p}{\rho_g \tau_v} k (1 - f_{vv}); \quad S_\varepsilon = 2 \frac{\rho_p}{\rho_g \tau_v} \varepsilon (1 - f_\varepsilon).$$

The coefficients of entrainment of a particle in the pulsation motion of the carrying turbulent flow f_{vv} , $f_{v\vartheta}$, and $f_{\vartheta v}$ are calculated by the formulas of [7] using an exponential approximation of the double-time correlation functions of turbulent pulsations of the velocity and temperature of the carrying flow along the trajectory of particle motion. The properties of the correlation function of the velocity and the temperature are taken to be the same [12]. The coefficient f is calculated based on the recommendations of [7, 8]. The difference between the scales of turbulent velocity pulsations in the coordinate directions is allowed for by the theory of locally homogeneous and locally isotropic turbulence [12].

3. Boundary Conditions. The predominant direction of flow development enables us to exclude the momentum equation in the projection onto the y axis from consideration and to ignore the pressure variation along the transverse coordinate. In the flow formed by injection, the transverse component of the velocity vector is much smaller in magnitude than the longitudinal velocity component and makes no perceptible contribution to the pressure distribution [6]. The indicated facts make it possible to use a parabolized formulation of the problem that does not require statement of the boundary conditions upstream.

The solution of Eqs. (1)-(6) must satisfy the boundary conditions on the wall and the axis of symmetry of the channel. On the channel axis, for $y = 0$, we prescribe the conditions of flow symmetry

$$v_g = \frac{\partial u_g}{\partial y} = \frac{\partial \vartheta_g}{\partial y} = \frac{\partial k}{\partial y} = \frac{\partial \varepsilon}{\partial y} = 0.$$

On a permeable wall, when $y = r_w$, conditions of normal injection are set for the components of the vector of gas velocity ($u_g = 0$, $v_g = -v_w$) while conditions that allow for the initial velocity nonequilibrium of the flow ($u_p = 0$, $v_p = -\varphi v_w$; $\varphi \leq 1$) are set for the dispersed phase. The wall temperature ($\vartheta_g = \vartheta_p = \vartheta_w$) is prescribed as a thermal boundary condition. The condition of the absence of velocity pulsations on the permeable surface of the channel is taken for the turbulence characteristic [6], so that $k = \varepsilon = 0$.

4. Transformation of the Equations. System of Eqs. (1)-(6) allows an order reduction. Let us assume that the distributions of the characteristics of the flow in different cross sections of the channel differ only in length and velocity scales:

$$u_g = u_m(x) U_g(y); \quad v_g = -v_w V_g(y); \quad \vartheta_g = \vartheta_w T_g(y);$$

$$u_p = u_m(x) U_p(y); \quad v_p = -v_w V_p(y); \quad \vartheta_p = \vartheta_w T_p(y);$$

$$k = u_m^2(x) K(y); \quad \varepsilon = u_m^3(x) E(y)/r_w.$$

Here u_m is the velocity on the channel axis. The half-width of the channel and the injection-rate modulus are used as the characteristic scales.

Eliminating the derivatives with respect to the longitudinal coordinate using the continuity equation, we reduce modeling of the flow to solution of a system of ordinary differential equations:

$$\frac{1}{y^n} (y^n V_g)' - U_g = 0; \quad (10)$$

$$\frac{1}{y^n} \left[\frac{y^n}{\text{Re}_m} (1 + \zeta) U_g' \right]' + M V_g U_g' - M U_g^2 = \frac{1}{\rho_g u_m^2} \frac{dp}{dx} + M B_v \frac{\rho_p}{\rho_g} (U_g - U_p); \quad (11)$$

$$\frac{1}{y^n} \left[\frac{y^n}{\text{Re}_m} \left(\frac{1}{\text{Pr}} + \frac{\zeta}{\text{Pr}_t} \right) T_g' \right]' + M V_g T_g' = M B_\delta \frac{\rho_p c_p^m}{\rho_g c_p} (T_g - T_p); \quad (12)$$

$$\frac{1}{y^n} \left[\frac{y^n}{\text{Re}_m} \left(1 + \frac{\zeta}{\sigma_k} \right) K' \right]' + M V_g K' - M U_g K = E + S_k - \frac{\zeta}{\text{Re}_m} U_g'^2; \quad (13)$$

$$\frac{1}{y^n} \left[\frac{y^n}{\text{Re}_m} \left(1 + \frac{\zeta}{\sigma_\varepsilon} \right) E' \right]' + M V_g E' - 3 M U_g E = c_{\varepsilon 2} \frac{E^2}{K} + S_\varepsilon - c_{\varepsilon 1} \frac{\zeta}{\text{Re}_m} \frac{E}{K} U_g'^2; \quad (14)$$

$$\frac{1}{y^n} (y^n V_p)' - U_p = 0; \quad (15)$$

$$M V_p U_p' - M U_p^2 = - \frac{1}{y^n} \left(\frac{y^n}{\text{Re}_m} \zeta f_{vv} U_g' \right)' - M B_v (U_g - U_p); \quad (16)$$

$$V_p V_p' = - \frac{2}{y^n} \left(\frac{y^n}{\text{Re}_m} \zeta f_{vv} V_g' \right)' - B_v (V_g - V_p); \quad (17)$$

$$M V_p T_p' = - \frac{1}{y^n} \left(\frac{y^n}{\text{Re}_m} \frac{\zeta}{\text{Pr}_t} f_{v\delta} T_g' \right)' - M B_\delta (T_g - T_p). \quad (18)$$

Here $\zeta = \nu_1/\nu$, $B_v \sim B_\delta = 1/\text{Stk}$. The solution depends on the parameter $M = 1/u_m$ and the Reynolds number $\text{Re}_w = \nu_m r_w/\nu$. The Reynolds number Re_m is constructed from the velocity at the channel axis u_m and is determined as $\text{Re}_m = \text{Re}_w/M$ in terms of the injection parameters. The dimensionless pressure gradient in Eq. (11) is found by targeting constancy of the flow rate through the channel cross section.

In new coordinates, the boundary conditions acquire the form:

$$\text{for } y = 0 \quad V_g = U_g' = T_g' = K' = E' = 0;$$

$$\text{for } y = 1 \quad U_g = U_p = 0, \quad V_g = T_g = T_p = 1, \quad V_p = \varphi, \quad K = E = 0.$$

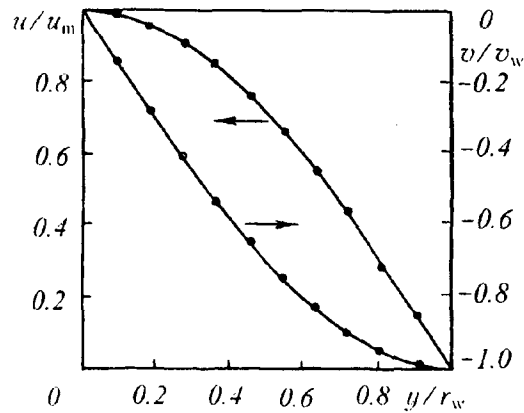


Fig. 2. Comparison of the calculated distributions of the components of the velocity vector (curves) with the data of a physical experiment [1] (points) for $Re_w = 10^5$.

To simplify the representation of Eqs. (10)-(18), we assume that $\rho_p = \text{const}$. This assumption, is consistent with the fact that solution of a system of equations in partial derivatives can be reduced to solution of a system of ordinary differential equations [9] and is allowed for comparatively simply within the framework of the approach developed.

It is assumed in constructing system of Eqs. (10)-(18) that the characteristics of the flow are functions of only one independent variable y . However, as the obtained equations show, their solution also depends implicitly on the variable x in terms of the parameter M . The latter is present only when the terms associated with turbulent viscosity are allowed for in the initial system of equations. For example, for laminar flow with particles, the parameter M is not involved in the system of equations obtained.

Numerical calculations performed for different parameters M showed that the dependence of the solution on it is rather weak. The terms that describe dissipative processes are much smaller in magnitude than the terms that model the generation of turbulence and convective transfer. Disregarding the indicated terms in the equations of the $k-\epsilon$ model, we have a system of equations independent of M . In [5], a model is constructed in which use is made of complete equations of the $k-\epsilon$ turbulence model, so it is possible to compare the results obtained with different approaches. Satisfactory agreement of the results is an argument that confirms the validity of the postulated form of solution.

5. Results of Numerical Modeling. System of Eqs. (10)-(18) was integrated numerically for different parameters M . The double-point boundary problem was solved based on scalar and vector runnings with the use of coefficient-matrix factorization and iterational coordination by nonlinearities and units of equations. To improve convergence, we used the method of lower relaxation. The pressure gradient was determined from the flow-rate relations.

Numerical results and experimental data on the velocity distribution in an infinite plane slot are compared in Fig. 2. As the injection intensity increases the influence of viscous effects on the structure of turbulent flow becomes weaker and manifests itself mainly in the axial region, leading to an insignificant filling of the velocity profile. In the infinite plane slot, the profile of the longitudinal component of the velocity vector becomes more extended and tends to a cosine distribution with strong injection as the Reynolds number increases ($Re_w \rightarrow \infty$). In axisymmetric flow, the distribution of the longitudinal velocity component becomes less filled as the injection intensity increases. For small Reynolds numbers, the profile of the longitudinal velocity component is described by the parabolic distribution that occurs in Poiseuille flow ($Re_w \rightarrow 0$).

The profiles of the transverse component of the velocity vector of the condensed phase differ comparatively slightly in a wide range of parameters. The effect of the initial nonequilibrium of the flow when the particle velocity on the wall differs from the injection rate leads to a deformed profile of the transverse component of the particle velocity near the mass-supply surface.

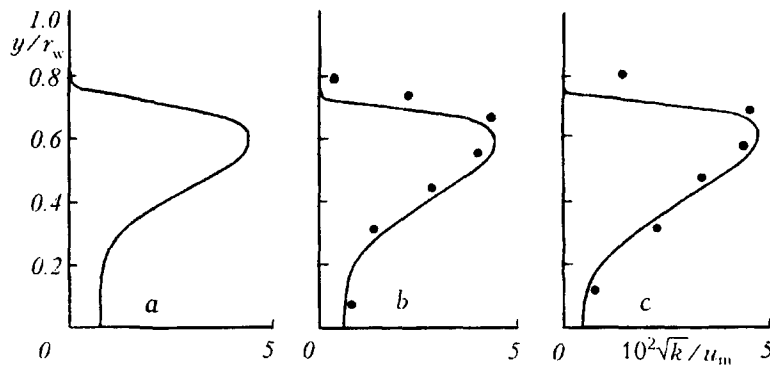


Fig. 3. Distributions of the kinetic turbulence energy in a channel with intense injection. The curves show the results of numerical modeling for $M = 0.020$ (a); 0.016 (b); 0.012 (c). The points show the data of [4, 5].

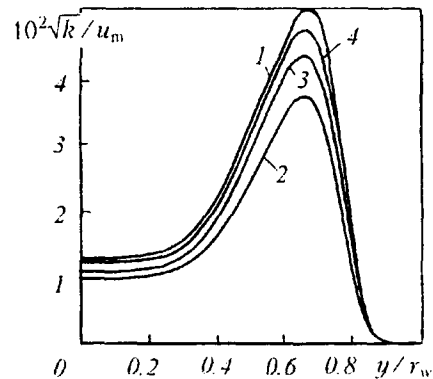


Fig. 4. Distributions of the kinetic turbulence energy in a channel with intense injection for $M = 0.036$ (the mass concentration of the impurity is 0.18): 1) flow in the absence of particles; 2) $d_p = 5 \mu\text{m}$; 3) 10 ; 4) 15 .

A characteristic of flow in a channel with a distributed mass supply is the presence of a negative pressure gradient due to acceleration of the flow because of the injection, which has a substantial effect on the mechanism and intensity of turbulent transfer. The distributions of pulsation characteristics of turbulence in a channel with permeable walls, as Fig. 3 shows, are in rather good agreement with the results of experimental measurements except for the region near the permeable wall. There is a good agreement as far as the maximum kinetic energy of turbulence in the cross section is concerned.

As the longitudinal coordinate increases the maximum of the kinetic turbulence energy shifts from the wall into the flow. A layer with vanishingly small values of turbulence energy is located near the permeable surface. A sharp increase in the level of turbulent velocity pulsations is observed in the region of a strong shift at a distance from the wall of the channel, where liquid particles moving along the normal to the surface are forced to turn around in a narrow surface zone. The dissipative function and hence the turbulent viscosity have similar characters of variation. The dependence of the distributions of the pulsation characteristics of turbulence on the Reynolds number is rather weak.

The presence of a discrete component has a laminarizing action on the flow. Profiles of the kinetic turbulence energy in two-phase flow are shown in Fig. 4.

The inverse effect of an impurity on the turbulence field is determined by the ratios of the time micro- and macroscales of turbulence in different regions of the flow to the relaxation time of a particle. The presence of two scales in the equations of the $k-\varepsilon$ turbulence model leads to a different character of action of the discrete component, depending on the inertia parameter of the impurity. The turbulizing action of very fine particle ($d_p \sim 1 \mu\text{m}$) that moves practically in equilibrium with the gas is due to a decrease in the viscous dissipation in the equation for the variable k . This is connected with the fact that fine particles, without interacting with energy-intensive pulsations of the gas, induce the suppression of the high-frequency part of the spectrum responsible for

the dissipation of turbulent energy. A decrease in the kinetic turbulence energy after the introduction of particles with diameter $d_p > 5 \mu\text{m}$ into the flow is explained by additional dissipation as a result of the interphase averaged and pulsation slip.

Conclusion. Comparison of the results of numerical modeling with experimental data shows that the approach developed, on the one hand, reproduces the pattern of flow quite well and allows for its basic features and, on the other, allows for a comparatively simple software realization with a reasonable combination of accuracy of the results produced and the computing time.

The method constructed for calculating flows in channels with permeable walls makes it possible to establish the extent to which the formation of the flow structure is affected by individual factors, and to make recommendations that facilitate the solution of the basic system of equations. The approach developed can be considered as a stage of transition to two- and three-dimensional models in the process of creating computational aids for refined modeling of flows in channels with injection.

NOTATION

x and y , Cartesian coordinates; u and v , components of the velocity vector; ρ , density; p , pressure; ϑ , temperature; k and ϵ , kinetic turbulence energy and its dissipation rate; c_p , specific heat of the gas at constant pressure; ν , kinematic viscosity; c_p^m , heat capacity of dispersed-phase material; d_p , particle diameter; τ_v and τ_θ , times of dynamic and thermal relaxations; r_w , half-width of the channel; v_w , injection-rate modulus; n , index of geometry of flow; Pr, Prandtl number; Re, Reynolds number; Stk, Stokes number. Subscripts and superscripts: g, gas; p, particle; m, channel axis; t, characteristics of turbulence; w, permeable wall.

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